# Solutions to JEE Advanced Home Practice Test -3 |JEE 2024 | Paper-2

## PHYSICS

**1.(AC)** Minima will be form at O.

If $SS_1 + S_1O - SS_2 - S_2O = \frac{n\lambda}{2}, n = 1, 3, 5, ......$

For minima value of d, $n = 1$

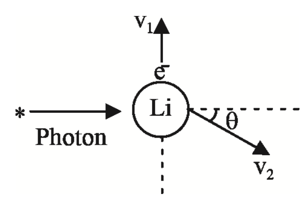
$\therefore \frac{\lambda}{4} = \sqrt{1+d^2} - 1$

$(1+d^2)^{1/2} - 1 = \frac{\lambda}{4}$

$1 + \frac{d^2}{2} - 1 = \frac{\lambda}{4}$ (Neglecting smaller terms)

$d = \pm\sqrt{\frac{\lambda}{2}}, \beta = \frac{d\lambda}{D} = \sqrt{2\lambda}$

**2.(BC)** Wavelength of incident photon is




$\lambda = \frac{12431}{5.4852} = 2266.28\,\text{Å}$

Photon momentum is $P = \frac{h}{\lambda}$

By conservation of energy we use

$\Delta E = \frac{1}{2}m_e v_1^2 + \frac{1}{2}m_{Li}v_2^2$ ........(i)

By conservation of momentum we use

$m_e v_1 = m_{Li} v_2 \sin\theta$ .......(ii)

And $\frac{h}{\lambda} = m_{Li} v_2 \cos\theta$ .......(iii)

Squaring adding (ii) and (iii) we get

$m_e^2 v_1^2 + \frac{h^2}{\lambda^2} = m_{Li}^2 v_2^2$

$\frac{1}{2}m_e v_1^2 = \frac{1}{2m_e}\left(m_{Li}^2 v_2^2 - \frac{h^2}{\lambda^2}\right)$

From equation (i) we use

$\Delta E = \frac{m_{Li}^2}{2m_e}v_2^2 - \frac{h^2}{2m_e\lambda^2} + \frac{1}{2}m_{Li}v_2^2$

$v_2 = \sqrt{\frac{\Delta E + \frac{h^2}{2m_e\lambda^2}}{\frac{m_{Li}}{2}\left(\frac{m_{Li}}{me}+1\right)}} = 14.2\,m/s$

From equation (iii) we use

$\cos\theta = \frac{h}{\lambda m_{Li} v_2} = 0.0178; \qquad \theta = 88.9°$

**3.(BD )** First of all the gas is compressed isothermally. Using Boyle's law

$P_1V_1 = P_2V_2$

Or $P_2 = (P_1V_1/V_2)$

Here $P_1 = 75\,cm$ of mercury and $V_2 = \frac{3}{4}V_1$

Thus $P_2 = \frac{75V_1}{(3/4)V_1} = 100\,cm$ of mercury.

The gas is now expanded adiabatically to $20\%$ greater of its original value. Under adiabatic changes the pressure and volume of gas are related as

$P_2V_2^{\gamma} = P_3V_3^{\gamma}$

Or $P_3 = P_2\left(\frac{V_2}{V_3}\right)^{\gamma}$

Here $V_2 = \frac{3}{4}V_1$ *and* $V_3 = \frac{120}{100}V_1$

Thus $P_3 = 100 \times \left(\frac{3V_1}{4}\right)^{1.5} \times \left(\frac{100}{120V_1}\right)^{1.5}$

$= 100 \times \left(\frac{3}{4}\right)^{1.5} \times \left(\frac{5}{6}\right)^{1.5} = 100 \times \left(\frac{5}{8}\right)^{1.5} = 100 \times 0.494 = 49.4$ cm of mercury.

Let the final temperature after adiabatic change be $T_3$ then from the relation of temperature and volume in an adiabatic process, we have

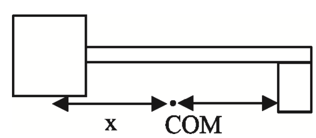
Now $T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

$T_2 = 17°C = 273 + 17 = 290\,K$

Now $T_3 = T_2\left(\frac{V_2}{V_3}\right)^{\gamma-1} = 290 \times \left(\frac{3V_1}{4}\right)^{1.5-1} \times \left(\frac{100}{120V_1}\right)^{1.5-1} = 290 \times \left(\frac{5}{8}\right)^{0.5} = 229.3K$

Hence the final temperature will be –43.7°C

**4.(AD)** $x = \frac{M.L.}{3M + M} = \frac{L}{4}$




From conservation of angular momentum about COM.

$Mv_0\left(\frac{3L}{4}\right) = I\omega$

$I = 3M\left(\frac{L}{4}\right)^2 + M\left(\frac{3L}{4}\right)^2 = \frac{3ML^2}{16} + \frac{9ML^2}{16} = \frac{3}{4}ML^2$

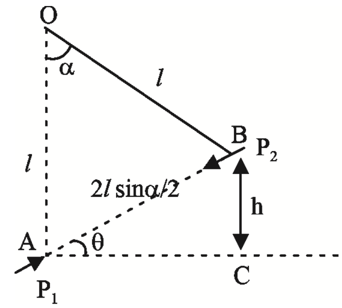
$\frac{3Mv_0L}{4} = \frac{3}{4}ML^2\omega$; $\quad \omega = \frac{v_0}{L}$

From cons. of linear momentum

$Mv_0 = 4M.v_c$; $v_c = \frac{v_0}{4}$

Velocity of 3M $= v_c - \frac{L}{4}\omega = \frac{v_0}{4} - \frac{L}{4} \times \frac{v_0}{L} = 0$

**5.(BD)** $U_i = 0$




$$U_f = \frac{2KP_1P_2}{\left[2l\sin\frac{\alpha}{2}\right]^3} + mgh \text{ ........(i)}$$

Now; form $\Delta AOB$

$\alpha + 90 - \theta + 90 - \theta = 180$

$\alpha = 2\theta$

$\Delta ABC : h = 2l\sin\left(\frac{\alpha}{2}\right)\sin\theta$

$$h = 2l\sin^2\left(\frac{\alpha}{2}\right); \qquad \frac{Mg}{\sin\left(90+\frac{\alpha}{2}\right)} = \frac{Fe}{\sin(180-2\theta)}$$

$$Fe = 2mg\sin\left(\frac{\alpha}{2}\right); \qquad \frac{6KP_1P_2}{\left(2l\sin\frac{\alpha}{2}\right)^4} = mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\frac{KP_1P_2}{\left(2l\sin\frac{\alpha}{2}\right)^3} = \frac{mg}{3}\sin\left(\frac{\alpha}{2}\right)\times\left(2l\sin\frac{\alpha}{2}\right) = \frac{mgh}{3}$$

$$U_f = \frac{2}{3}mgh + mgh = \frac{5}{3}mgh$$

**6.(AD)** The rate of collision of the molecules with per square meter of the wall is $(1/6)n_0v$ where $n_0$ is the molecular density and v is RMS speed of molecules and pressure exerted by the gas on wall is given by $\left(\frac{1}{6}\right)n_0v\times 2m'v$ where $m'$ is the mass of each molecule.

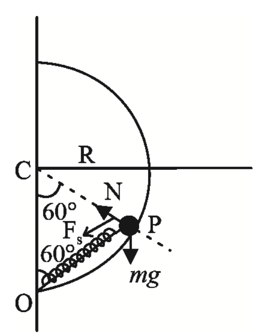
**7.(AD)** For lens $L_1$, ray must move parallel to the axis after refraction $\frac{\mu_1}{\infty} + \frac{\mu_w}{X} = \frac{\mu_1 - \mu_w}{R_1} \Rightarrow x = 10\,cm$

For lens $L_2$, image must form at centre of curvature of the curved surface after refraction through plane part.

$$\frac{\mu_2}{-R_2} + \frac{\mu_\omega}{x'} = 0;\ x' = 8\,cm$$

**8.(BC)** Spring force $F_s = k\left(\frac{R}{4}\right) = \frac{mg}{4}$

Equation of motion in tangential direction is




$$mg\cos 30° + \frac{mg}{4}\cos 30° = ma$$

$$mg\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8}\right) = ma; \qquad a = \frac{5\sqrt{3}g}{8} = \frac{25\sqrt{3}}{4}m/s^2$$

Along radial direction

$$N + \frac{mg}{4}\sin 30° = mg\sin 30°;\ N = \frac{mg}{2} - \frac{mg}{8} = \frac{3mg}{8} = \frac{30}{8} = 3.75N$$

**9.(8.4)** EMF developed across the rod = $\int_1^4 \frac{\mu_0 i}{2\pi r} drv = \frac{\mu_0 iv}{2\pi} \ln 4 = \frac{\mu_0 iv}{\pi} \ln 2$

From given value, $E = \frac{4\pi \times 10^{-7} \times 2 \times 3 \times 0.7}{\pi} = 24 \times 7 \times 10^{-8}$

$i_{\max} = \frac{E}{R} = \frac{24 \times 7 \times 10^{-8}}{1.4} = 1.2 \times 10^{-6} A$

$Q_{\max} = C_0 E = 24 \times 7 \times 10^{-8} \times 5 \times 10^{-6} = 8.4 \times 10^{-12} C$

**10.(0.81-0.82).**

Energy of daughter nuclei in two cases of $\alpha$-emission are $E_{D_1} = \frac{4}{206} \times E_{\alpha_1} = \frac{4}{206} \times 5.3\, MeV$

$E_{D_1} = 0.1029\, MeV$

$E_{D_2} = \frac{4}{206} \times E_{\alpha_2} = \frac{4}{206} \times 4.5\, MeV$

$E_{D_2} = 0.0873\, MeV$

Thus total energy released in first case reaction is

$E_T = E_{D_1} + E_{\alpha_1} = 5.3 + 0.1029 MeV = 5.4029\, MeV$

In second case when $\gamma -$ photon is released, we use

$E_T = E_{D_1} + E_{\alpha_1} = E_{D_2} + E_{\alpha_2} + E_\gamma$

$E_\gamma = E_T - E_{D_2} - E_{\alpha_2} = 5.40 - 4.50 - 0.09 = 0.81 MeV$

**11.(100)** As 30% light is incident and reflected by mirror, force due to reflection of light on mirror is

$F = 2 \times \frac{0.3P}{c} = \frac{0.6P}{c}$

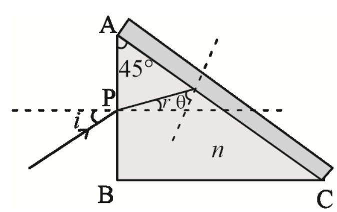
To support the weight of mirror, we use

$\frac{0.6P}{c} = mg$ ; $P = \frac{mgc}{0.6} = \frac{20 \times 10^{-3} \times 10 \times 3 \times 10^8}{0.6}$

$P = 10^8$ watt ; $P = 100 \times 10^6 W = 100\, MW$

**12. (0.60 - 0.63)**

Let us consider the critical angle for face AC be $\theta_c$ by snell's law




$\sin \theta_c = \left( \frac{n_1}{n} \right)$

From figure.

$r + \theta_c = 45°$ ; $r = 45° - \theta_c$

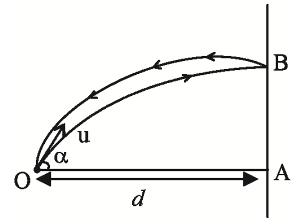
For refraction at '$p$'

$1 \times \sin i = n \sin r$

$\sin i = n \sin\left[45° - \theta_c\right] = n \sin\left[45° - \sin^{-1}\left(\frac{n_1}{n}\right)\right]$

$= n \sin\left[45° - \sin^{-1}\left(\frac{1.2}{2.4}\right)\right] = n \sin\left[45° - 30°\right] = n \sin 15° = 2.4 \sin 15° = 2.4 \times 0.258$ ; $\sin i = 0.62$

**13.(2)** The only vertical force on the ball is mg throughout its motion because during impact it experiences a horizontal force from the wall. We can use




$$u_y t - \frac{1}{2}gt^2 = s_y$$

Let $t$ be the total time of flight.

$$\therefore \quad O = u\sin\alpha t - \frac{1}{2}gt^2; \qquad t = \frac{2u\sin\alpha}{g}$$

Due to impact with the wall at B, the normal component (i.e., horizontal component) of velocity is reversed and become $e$ times.

Horizontal velocity before impact $= u\cos\alpha$

And horizontal velocity after impact $= eu\cos\alpha$

Time taken to reach the wall, $t_1 = \dfrac{d}{u\cos\alpha}$

And time taken to come back to O from B.

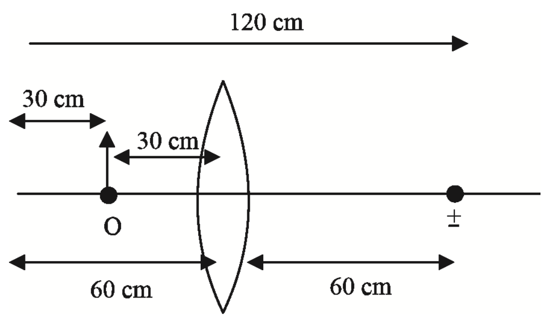
$$t_2 = \frac{d}{u\cos\alpha} + \frac{d}{eu\cos\alpha} = \frac{2u\sin\alpha}{g} \Rightarrow u^2\sin 2\alpha = gd\left[1+\frac{1}{e}\right]$$

As $\sin 2\alpha \le 1$,

$$\frac{gd}{u^2}\left[1+\frac{1}{e}\right] \le 1 \Rightarrow d \le \frac{eu^2}{g(1+e)}$$

$$d \le \frac{2eu^2}{g(2+2e)}; \qquad x = 2$$

**14.(1.11)**




Least count of scale of optical bench $= \dfrac{1\,cm}{5} = 0.20\,cm$

$$u = -(x_2 - x_1)$$

$$\Delta u = \Delta x_1 + \Delta x_2 = 0.4\,cm; \qquad \Delta v = 0.4\,cm$$

$$u = -30\,cm;\ v = +60\,cm$$

$$f = \frac{uv}{u-v} = \frac{-30\times 60}{-30-60} = 20\,cm; \qquad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}(lens\ formula)$$

$$-\frac{df}{f^2} = -\frac{dv}{v^2} + \frac{du}{u^2}; \qquad \frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\frac{\Delta f}{f}\times 100 = f\times 100\left[\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}\right] = 20\times 100\left[\frac{0.4}{60\times 60}+\frac{0.4}{30\times 30}\right] = 20\times 100\times 0.4\left[\frac{1+4}{3600}\right] = 1.11$$

**15.(C)** $W_{gas} = nR\Delta T$ (isobaric) $= 2\times\frac{25}{3}\times 60 = 1000J$

$\Delta U = \frac{f}{2}nR\Delta T = \frac{3}{2}\times 2\times\frac{25}{3}\times 60 = 1500\,J$; $Q = w + \Delta U = 2500J$

Also, from work kinetic energy theorem on piston

Work done by gravity = Work done by gas = 1000 J

**16.(D)** Piston is fixed (isochoric); $W_{gas} = 0$; $\Delta U_g = \frac{f}{2}nR\Delta T = \frac{3}{2}\times 2\times\frac{25}{3}\times 60 = 1500J$

$Q = W + \Delta U$; $Wg = mgh = 0$

**17.(C)** **Case I**

$M = \frac{m}{2}, L = 2l_0, \mu = \frac{\mu_0}{4}$; $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(2l_0)}\sqrt{\frac{mg}{2\times\frac{\mu_0}{4}}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

**Case-II**

$M = 2m, L = 2l_0, \mu = \frac{\mu_0}{2}$; $f_2 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(2l_0)}\sqrt{\frac{2mg}{\frac{\mu_0}{2}}} = \frac{2}{2} = 1$

**Case III**

$M = \frac{m}{4}, L = 3l_0, \mu = \frac{\mu_0}{4}$; $f_3 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(3l_0)}\sqrt{\frac{mg}{4\times\frac{\mu_0}{4}}} = \frac{1}{3}$

**Case IV**

$M = \frac{m}{8}, L = 4l_0, \mu = \frac{\mu_0}{32}$; $f_4 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(4l_0)}\sqrt{\frac{mg}{8\times\frac{\mu_0}{32}}} = \frac{2}{4} = \frac{1}{2}$

**18.(B)** **Case I**

$M = \frac{m}{2}, L = 2l_0, \mu = \frac{\mu_0}{4}$; $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(2l)}\sqrt{\frac{mg}{2\times\frac{\mu_0}{4}}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

**Case II**

$M = 2m,\ L = 2l_0,\ \mu = \frac{\mu_0}{2}$

$2^{nd}$ harmonic ; $f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}} = \frac{2}{2(2l_0)}\sqrt{\frac{2mg}{\mu_0/2}} = 2$

**Case –III**

$M = \frac{m}{4}, L = 3l_0, \mu = \frac{\mu_0}{4}$; $3^{rd}$ harmonic frequency

$f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}} = \frac{3}{2(3l_0)}\sqrt{\frac{Mg}{4\times\frac{\mu_0}{4}}} = 1$

**Case IV**

$M = \frac{m}{8},\ L = 4l_0, \mu = \frac{\mu_0}{32}$

$6^{th}$ harmonic frequency ; $f_4 = \frac{6}{2L}\sqrt{\frac{T}{\mu}} = \frac{6}{2(4l_0)}\sqrt{\frac{mg}{8\times\frac{\mu_0}{32}}} = 3$

## CHEMISTRY

**19.(AD)(A)** $Na_2SO_3 + S \xrightarrow[OH^- \text{ or } H_2O]{\text{boiling}} Na_2S_2O_3$

**(B)** $S_2O_3^{2-}$ formed in presence of acid precipitates sulphur and liberates $SO_2$ gas according to the following reaction.

$$S_2O_3^{2-} + H^+ \xrightarrow[\text{disproportionation}]{} S\downarrow(\text{white}) + SO_2\uparrow + H_2O$$

**(C)** $2Ca_3(PO_4)_2 + 6SiO_2 + 10C \xrightarrow{1400-1500°C} 6CaSiO_3 + 10CO + P_4$

**(D)** $4AgNO_3 + 2H_2O + H_3PO_2 \longrightarrow 4Ag + 4HNO_3 + H_3PO_4$

**20.(ABD)** As bottle 2 + bottle 3 gives colourless and odourless gas, it may be carbon dioxide. Generally carbonates are decomposed by acids giving $CO_2$ gas. It suggests that bottle 2 and 3 contain sodium carbonate and HCl. Bottle 3 + 4 gives blue precipitate which confirms the $Cu^{2+}$ in either of bottles. $CuSO_4, CuCl_2$ and $Cu(NO_3)_2$ are soluble and $CuCO_3$ is insoluble in water as evident from the reaction.

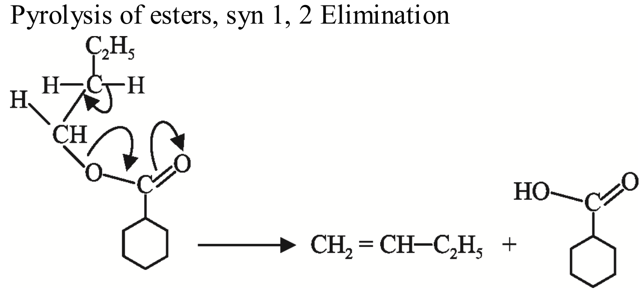
$Cu^{2+} + CO_3^{2-} \rightarrow CuCO_3\downarrow(\text{blue})$. Thus blue precipitate must be of copper carbonate.

Hence, bottle 4 is $CuSO_4$, 3 is $Na_2CO_3$, 2 is HCl (from above) and 1 is $Pb(NO_3)_2$ as it gives white precipitate of $PbCl_2$ with bottle (2).

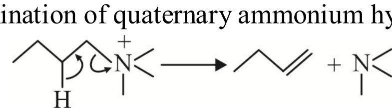
Bottle 4 is $CuSO_4$ and that gives deep blue colouration with excess of ammonia solution.

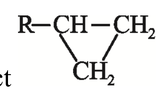
$Cu^{2+}(aq) + 4NH_3(aq) \rightarrow [Cu(NH_3)_4]^{2+}$ (deep blue colour) (aq.)

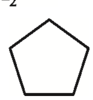
**21.(AD) (I)** Pyrolysis of esters, syn 1, 2 Elimination

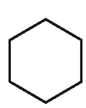



**(II)** Hoffmann elimination of quaternary ammonium hydroxide

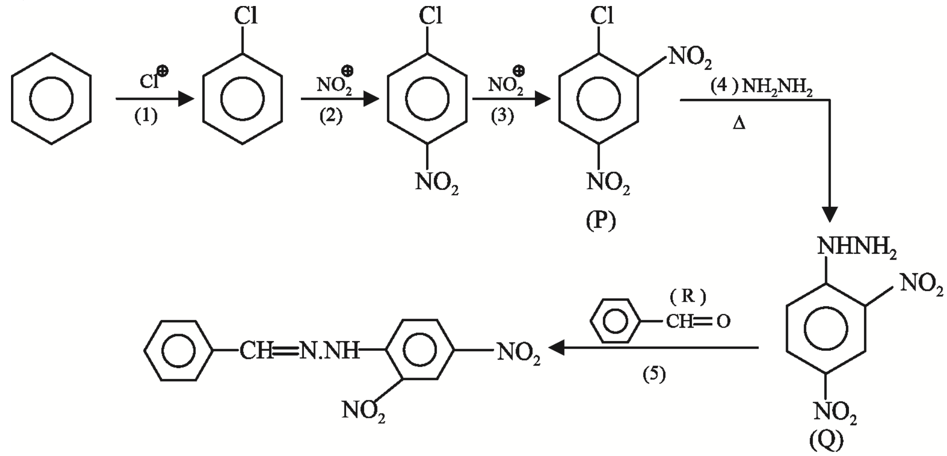



**(III)** Product 


**(IV)** Product  (Wurtz reaction)

**(V)** Product  (Decarboxylation reaction)

**22.(BCD)**



**23.(ACD)** P and Q are $Na_2CrO_4, H_2SO_4$

X and Y are Carbon and Aluminium

$Fe_2O_3$ gets precipitated as $Fe(OH)_3$ on dissolving in $H_2O$

**24.(BD)** (B) Probability of finding electron between distance r and

$r + dr = \Psi^2 .4\pi r^2 dr = \left(\frac{Z^3}{\pi a_0^3}\right)(4\pi r^2)e^{-2Z/a_0}dr$ For maximum $f(r) = \left(\frac{Z^3}{\pi a_0^3}\right)(4\pi r^2)e^{-2Z/a_0}$

must be maximum.

$$\text{Setting } \frac{df(r)}{dr} = 0 \Rightarrow 2r = r^2\left(\frac{2Z}{a_0}\right) \Rightarrow r = \frac{a_0}{Z} \;;\; (a_0 = 52.9\,\text{pm})$$

For $He^+$ ion $= \frac{52.9}{2} = 26.45$ pm is the most probable distance of electron in $He^+$ ion.

(C) $\Psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/a_0} = 0$

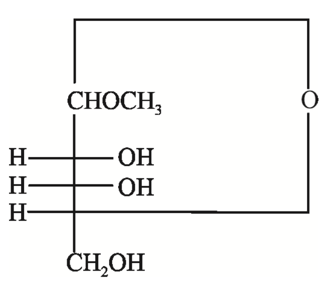
At radial node, $\Psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/a_0} = 0$

$\left(2 - \frac{r}{a_0}\right) = 0 \Rightarrow r = 2a_0$

(D) No. of spherical nodes $= n - l - 1$;

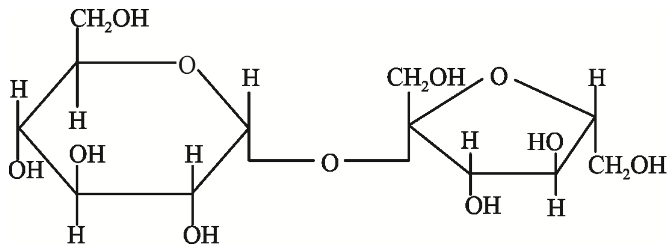
**25.(BD)**

**(B)**



The number of moles of $HIO_4$ required to break down the above molecule is 1

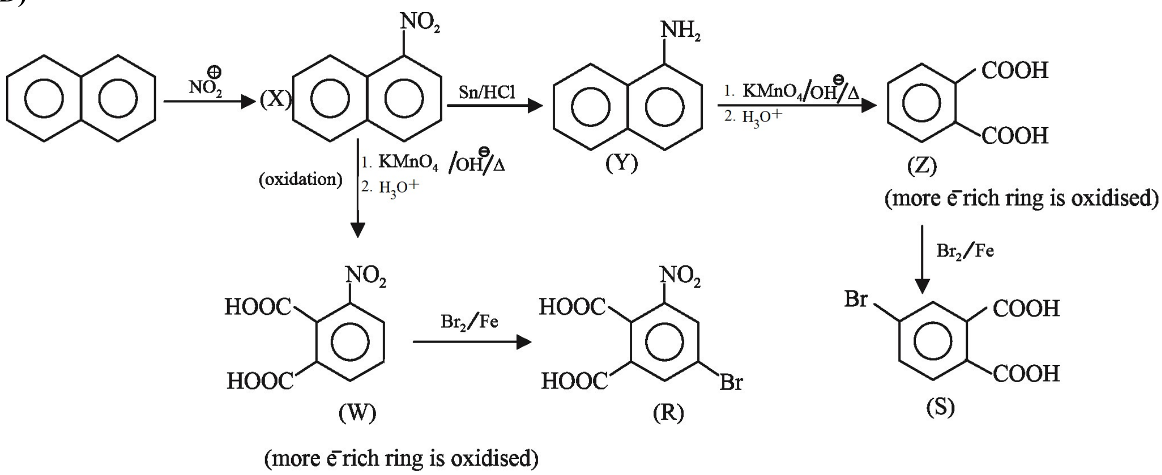
**(C)**




The compound is sucrose which on hydrolysis gives equimolecular mixture of glucose and fructose.

**(D)** Polyester is formed by condensation of diacids with diols.

**26.(ABD)**

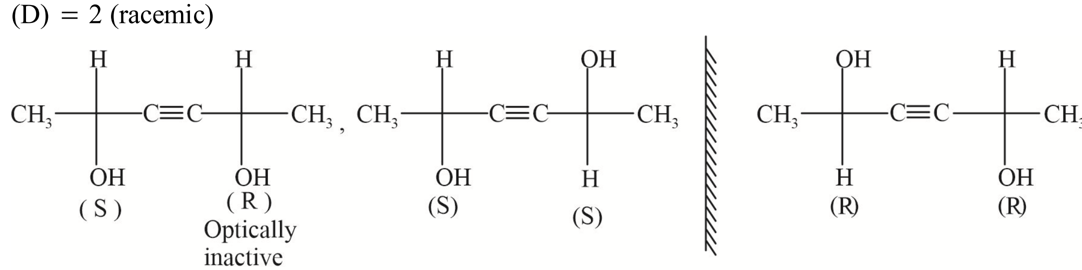



**27.(8)** (A) = 3

(B) = 2 (optically active-2; optically inactive-1)

(C) = 1 (optically active -2; optically inactive -1)

(D) = 2 (racemic)

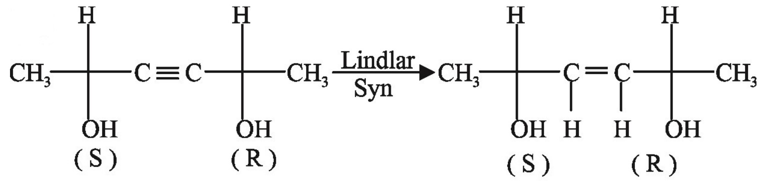


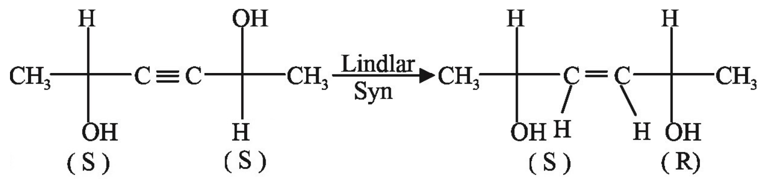

Enantiomers optically active

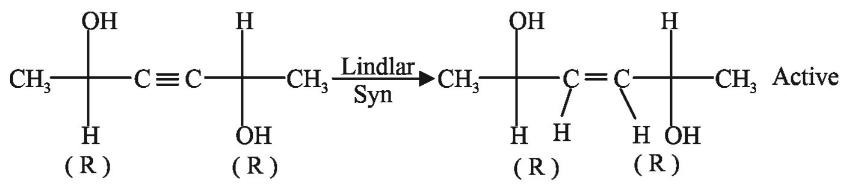
Total stereoisomers -3

Optically active-2

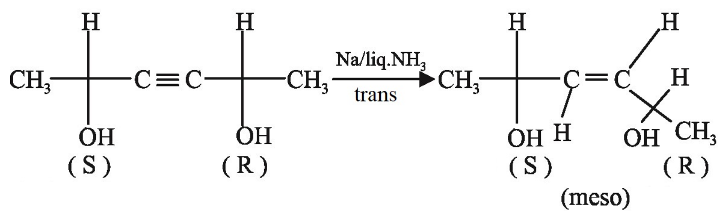
Optically inactive-1

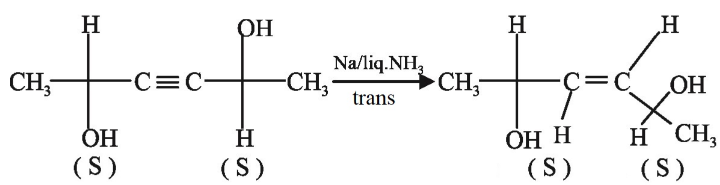


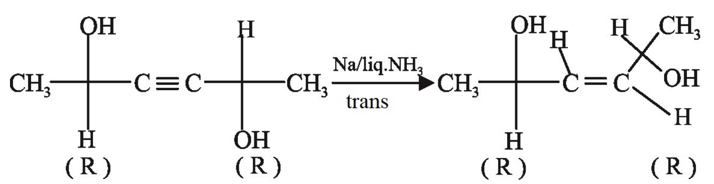




Total stereoisomer -3
optically active-2
optically inactive-1

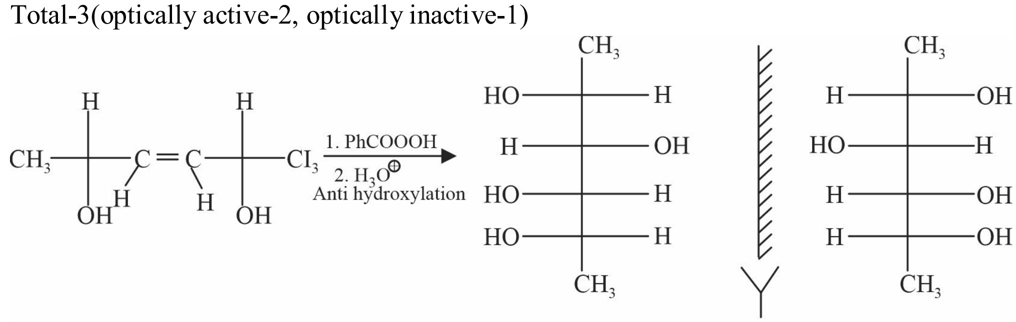



Both R, R and S, S on reduction will give one stereoisomer optically active.

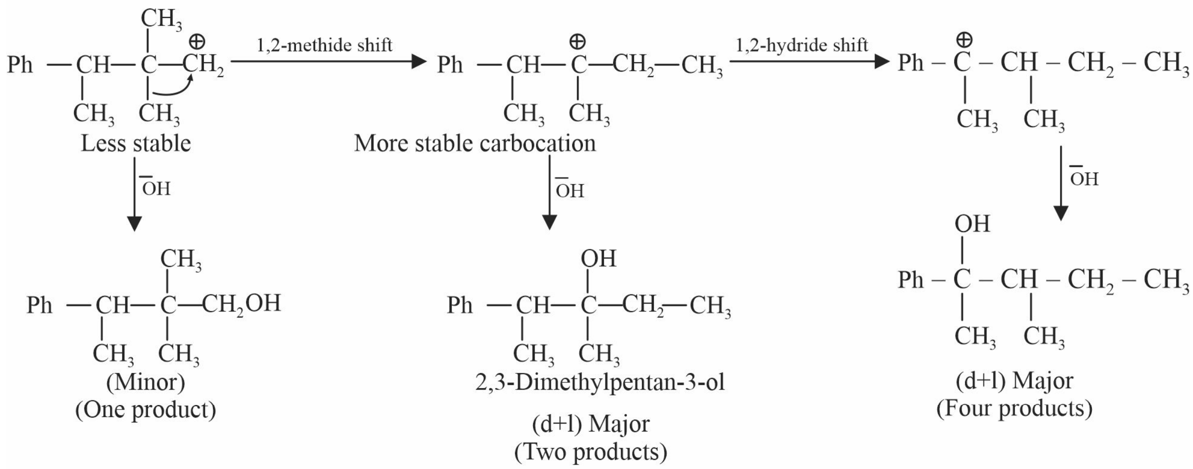
Total-3(optically active-2, optically inactive-1)

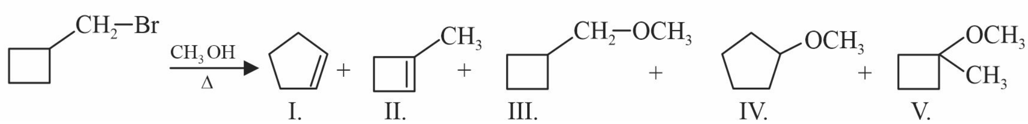



racemic mixture

**28.(9)**

X = 7
Y = 2




**29.(61.25)** $5H_2SO_4 + 8NaI \longrightarrow 4Na_2SO_4 + 4I_2 + H_2S + 4H_2O$

n-factor of $H_2SO_4 = \frac{8}{5}$

Eq. wt $= \frac{\text{mol.wt}}{\text{n}} = \frac{98 \times 5}{8} \Rightarrow 61.25$

**30.(36)**

**X = 6**

Complex ion is $\left[Fe(H_2O)_5 SCN\right]^{2+}$; it has coordination number six.

Y = 1

The O.N. of Fe in $\left[\overset{X}{Fe}\left(\overset{0}{H_2O}\right)_5 \overset{+1}{NO}\right]^{2+} SO_4^{2-}$ is $x + 0 + 1 = +2$ or $x = +1$

Z = 6

$\left[Mn(CN)_6\right]^{4-} - 3d^5, d^2sp^3; +2$ oxidation state

$\left[Ni(NH_3)_6\right]^{2+} - 3d^8, sp^3d^2; +2$ oxidation state

$\left[Co(ox)_3\right]^{3-} - 3d^6, d^2sp^3; +3$ oxidation state

$\left[Cu(NO_2)_6\right]^{4-} - 3d^9, sp^3d^2; +2$ oxidation state

$\left[AgF_4\right]^{-} - 4d^8, dsp^2; +3$ oxidation state

$\left[Ni(CN)_4\right]^{2-} - 3d^8, dsp^2; +2$ oxidation state

$\left[PdCl_4\right]^{2-} - 4d^8, dsp^2; +2$ oxidation state

$\left[Pd\left(CN\right)_4\right]^{2-} - 4d^8, dsp^2; +2$ oxidation state

$\left[Co\left(SCN\right)_4\right]^{2-} - 3d^7, sp^3; +2$ oxidation state

**31.(120)** $A(g) \rightarrow \frac{2}{3}B(g) + \frac{2}{3}C(g)$

$t = 0 \quad P_0$

$t = 20 \quad P_0 - x \qquad \frac{2x}{3} \qquad \frac{2x}{3}$

$t = \infty \qquad \frac{2P_0}{3} \qquad \frac{2P_0}{3}$

$\frac{4P_0}{3} = 4; \qquad P_0 = 3\,atm$

$P_0 + \frac{x}{3} = 3.5 \quad ; \qquad x = 1.5$

for first order kinetics

$\Rightarrow \ln\frac{P_0}{P_0 - x} = kt \qquad \Rightarrow \frac{\ln 2}{k} = 20$

$t_{50\%} = 20$ is the half life

$t_{75\%} = 2 \times 20 = 40\,min$

$t_{87.5\%} = 3 \times t_{50\%} = 3 \times 20 = 60\,min$

**32.(32)** I. 40 g, O combines with 60 g metal
8 g O combines with 12 g metal
$X = 12$

II. $29.2\%$ (w/w) HCl has density = 1.25 g/ml
Now, mole of HCl required in 0.4 M HCl (500 ml)
$= (0.4 \times 0.5)\,mole = 0.2\,mole$
If V ml of original HCl solution is taken, then
mass of solution $= 1.25V$

Mass of HCl $= \left(1.25V \times 0.292\right)$; Mole of HCl $= \frac{1.25V \times 0.292}{36.5} = 0.2$

So, $V = \frac{36.5 \times 0.2}{0.292 \times 1.25}\,mol = 20\,mL$

**33.(B)**

**34.(C)** $\left(I \rightarrow R, S\right); \left(II \rightarrow P, S\right); \left(III \rightarrow Q, R, S\right); \left(IV \rightarrow P, Q, S, T\right)$

(A) $6 \rightarrow 3 \qquad \Delta n = 3$

No. of lines $= \frac{3\left(3+1\right)}{2} = 6$

All lines are in infrared region

(B) $7 \rightarrow 3 \qquad \Delta n = 4$

No. of lines $= \frac{4\left(4+1\right)}{2} = 10$

All lines are in infrared region

(C) $5 \rightarrow 2 \quad \Delta n = 3$

Lines are in visible region and also in infrared region

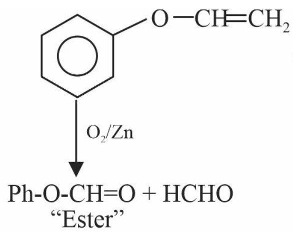
(D) $5 \rightarrow 1$ Number of lines $= \dfrac{5 \times 4}{2} = 10$

Lines are in visible region, in infrared region and also in U.V. region.

**35.(A)**

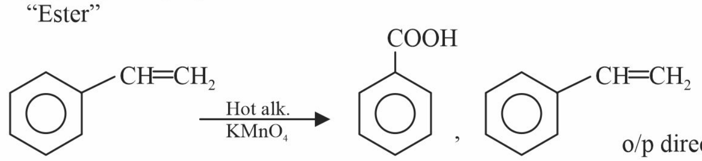
**36.(B)** $(I) \rightarrow q, r, s; (II) \rightarrow p, q; (III) \rightarrow p, q; (IV) \rightarrow p, t$

A.



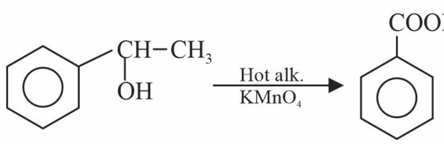

o/p directing due to +m effect of -O-CH=$CH_2$

B.




o/p directing due to +m effect of CH=$CH_2$

C.




D. is meta directing due to -m effect of -$COCH_3$ group

## MATHEMATICS

**37.(ACD)** $Y=\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}$ By partial fraction decomposition

$$=\int_0^1\left(\frac{2}{(x+1)}-\frac{1}{(x^2+2x+2)}\right)dx=\int_0^1\left(\frac{2}{(x+1)}-\frac{1}{(x+1)^2+1}\right)dx$$

$$=2\ln(x+1)-\arctan(x+1)\Big|_0^1$$

$$=2\ln 2-\arctan 2+\frac{\pi}{4}\text{.......(A)}$$

From (A)

$$Y=2\ln 2+\frac{\pi}{4}-\arctan 2=2\ln 2+\arctan\left(\frac{1-2}{1+2}\right)=2\ln 2-\text{arccot}\,3\text{ .....(C)}$$

From (A)

$$Y=2\ln 2+\frac{\pi}{4}-\arctan 2=-\frac{\pi}{4}+\ln 4+\frac{\pi}{2}-\arctan 2=-\frac{\pi}{4}+2\ln 2+\text{arccot}\,2\text{ ......(D)}$$

**38.(BD)** Shift LHS integral to RHS to get

$$\int_2^{150}\left(f^2(x)-(x-1)\ln(x-1)\left(2f(x)-(x-1)\ln(x-1)\right)\right)dx=0$$

$$\Rightarrow\int_2^{150}\left(f^2(x)-2f(x)(x-1)\ln(x-1)+\left((x-1)\ln(x-1)\right)^2\right)dx=0$$

$$\Rightarrow\int_2^{150}\left(f(x)-(x-1)\ln(x-1)\right)^2dx=0$$

But, $\left(f(x)-(x-1)\ln(x-1)\right)^2\ge 0$

So, $\left(f(x)-(x-1)\ln(x-1)\right)^2=0$

$f(x)-(x-1)\ln(x-1)=0$

$f(x)=(x-1)\ln(x-1)$

(A) is incorrect because area is equal to ¼ and check other options.

**39.(ABD)** $m_1^2=\frac{2b^2+2c^2-a^2}{4}$; $m_2^2=\frac{2c^2+2a^2-b^2}{4}$; $m_3^2=\frac{2a^2+2b^2-c^2}{4}$;

$$\therefore\quad \begin{bmatrix}m_1^2\\m_2^2\\m_3^2\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{bmatrix}\begin{bmatrix}a^2\\b^2\\c^2\end{bmatrix};\qquad \begin{bmatrix}m_1^2\\m_2^2\\m_3^2\end{bmatrix}=\frac{A}{4}\begin{bmatrix}a^2\\b^2\\c^2\end{bmatrix}\Rightarrow 4A^{-1}\begin{bmatrix}m_1^2\\m_2^2\\m_3^2\end{bmatrix}=\begin{bmatrix}a^2\\b^2\\c^2\end{bmatrix}$$

$$\begin{bmatrix}a^2\\b^2\\c^2\end{bmatrix}=\frac{4}{9}\begin{bmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{bmatrix}\begin{bmatrix}m_1^2\\m_2^2\\m_3^2\end{bmatrix};\qquad M=\frac{4}{9}\begin{bmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{bmatrix}$$

Now, verify.

**40.(ABD)** Let $f(x)=ax^3+bx^2+cx+d$

$$\lim_{x\to 0}\left(1+f(x)\right)^{\frac{1}{x}}=e^{-1}\Rightarrow c=-1\, and\, d=0$$

$$x^3 f\left(\frac{1}{x}\right)=x^3\left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{c}{x}+d\right)=a+bx+cx^2+dx^3$$

$$\lim_{x\to 0}\left(x^3 f\left(\frac{1}{x}\right)\right)^{1/x}=e^2\Rightarrow \lim_{x\to 0}\left(a+bx+cx^2+dx^3\right)^{1/x}=e^2$$

$a=1$ and $b=2$

$f(x)=x^3+2x^2-x$

**41.(ABD)** $f(x)=a(x-1)^2-2$

$$\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}\left[a(x-1)^2-2\right]=3$$

$a-2=3\Rightarrow a=5$

$g:[1,\infty)\to[-2,\infty)$

$g(x)=5(x-1)^2-2$

A. $g'(x)=10(x-1)\Rightarrow g'(1)=0$

B. Domain of $g(g(x))$

$g(x)\ge 1\Rightarrow 5(x-1)^2-2\ge 1$

$$x\ge 1+\sqrt{\frac{3}{5}}$$

$$\therefore x\in\left[1+\sqrt{\frac{3}{5}},\infty\right)=\left[1+\sqrt{\frac{p}{q}},\infty\right)$$

$q-p=2$

(C) $g(x)=g^{-1}(x)=x$

$5(x^2-2x+1)-2=x\Rightarrow 5x^2-11x+3=0$

$$x=\frac{11\pm\sqrt{121-60}}{10}$$

$$x=\frac{11\pm\sqrt{61}}{10}\begin{cases}\nearrow\frac{11+\sqrt{61}}{10}(only\, one\, solution)\\ \searrow\frac{11-\sqrt{61}}{10}(rejected)\end{cases}$$

(D) $$\frac{d}{dx}\left[90\left(g^{-1}(x)\right)\right]\Big|_{x=43}=\frac{90}{g'(4)}=\frac{90}{10(3)}=3$$

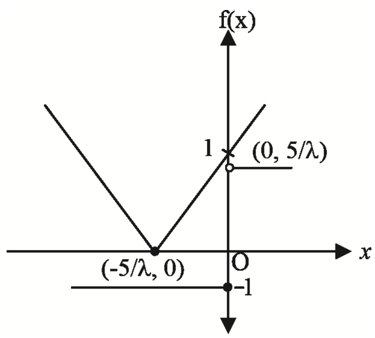
$g(x)=43\Rightarrow 5(x-1)^2-2=43\Rightarrow x-1=3\Rightarrow x=4$

**42.(ACD)** $f(x)=\lim_{n\to\infty}(-n)\left(\left|2\tan^{-1}x-\frac{1}{n}\right|-2\left|\tan^{-1}x\right|\right)$

$$=\lim_{n\to\infty}\frac{(-n)\left(\left(2\tan^{-1}x-\frac{1}{n}\right)^2-4\left(\tan^{-1}x\right)^2\right)}{\left|2\tan^{-1}x-\frac{1}{n}\right|+2\left|\tan^{-1}x\right|}$$

$$=\lim_{n\to\infty}\frac{(-n)\left(\frac{-4\tan^{-1}x}{n}+\frac{1}{n^2}\right)}{\left|2\tan^{-1}x-\frac{1}{n}\right|+2\left|\tan^{-1}x\right|}=\frac{4\tan^{-1}x}{\left|4\tan^{-1}x\right|}=\frac{\tan^{-1}x}{\left|\tan^{-1}x\right|},x\neq 0$$

$$f(x)=\begin{cases}\frac{\tan^{-1}x}{\left|\tan^{-1}x\right|}, & x\neq 0\\ -1, & x=0\end{cases}$$




(A) $f(x)$ is discontinuous at $x=0$

(B) $|f(x)|$ is a continuous functions.

(C) $f(1)+f(2)=2$

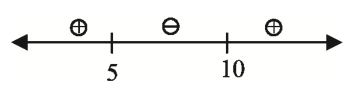
(D) $f(x)=\left|x+\frac{5}{\lambda}\right|$

For the existence of the solution of the equation $\frac{5}{\lambda}<1\Rightarrow\lambda>5$

**43.(ABCD)**

$\lim_{x\to\infty}\cot^{-1}x=0$ and $\lim_{x\to-\infty}\cot^{-1}x=\pi$

And $(x-5)(x-10)$




**44.(AB)** $\frac{d}{dx}(P(x))+(x-1)^3-(P(x)+1)\geq 0$

$$e^{-x}\left(\frac{d}{dx}(P(x))-P(x)+x^3-3x^2+3x-2\right)\geq 0$$

$$\left(\frac{d}{dx}\left(P(x)e^{-x}\right)-\frac{d}{dx}e^{-x}x^3-3\frac{d}{dx}xe^{-x}-\frac{d}{dx}e^{-x}\right)\geq 0$$

$$\frac{d}{dx}\left(P(x)-\left(x^3+3x+1\right)e^{-x}\right)\geq 0$$

Let $g(x)=\left(P(x)-\left(x^3+3x+1\right)\right)e^{-x}$ is increasing

$$g(x)\geq g(0)\Rightarrow\left(P(x)-\left(x^3+3x+1\right)\right)e^{-x}\geq 0\,\forall x\geq 0$$

But $P(x)\leq x^3+3x+1\ \forall\ x\geq 0$

$$P(x)=x^3+3x+1\ \forall\ x\geq 0$$

**45. (1.57)** $\sum_{\omega=1}^{\infty}\sin^{-1}\left[\frac{2\omega+1}{\omega(\omega+1)\left(\sqrt{\omega^2+2\omega}+\sqrt{\omega^2-1}\right)}\right]=\sum\sin^{-1}\left[\frac{(2\omega+1)\left(\sqrt{\omega^2+2\omega}-\sqrt{\omega^2-1}\right)}{\omega(\omega+1)(2\omega+1)}\right]$

$$=\sum\sin^{-1}\left[\frac{\left(\sqrt{(\omega+1)^2-1}-\sqrt{\omega^2-1}\right)}{\omega(\omega+1)}\right]$$

$$\sum_{\omega=1}^{\infty}\sin^{-1}\left[\frac{1}{\omega}\sqrt{1-\frac{1}{(\omega+1)^2}}-\frac{1}{\omega+1}\sqrt{1-\frac{1}{\omega^2}}\right];\qquad\sum_{\omega=1}^{\infty}\left(\sin^{-1}\frac{1}{\omega}-\sin^{-1}\frac{1}{\omega+1}\right)$$

$$S_n=\sin^{-1}1-\sin^{-1}\frac{1}{2}+\sin^{-1}\frac{1}{2}-\sin^{-1}\frac{1}{3}+.....+\sin^{-1}\frac{1}{n}-\sin^{-1}\frac{1}{n+1}$$

$$S_n=\sin^{-1}1-\sin^{-1}\frac{1}{n+1};\ S_\infty=\frac{\pi}{2}$$

**46.(30)** Rewrite the integral as

$$I_2=\int_0^1\left(\frac{x}{5+x}\right)^{7/2}\left(\frac{1-x}{5+x}\right)^{9/2}\frac{dx}{(5+x)^2}$$

And do the substitution $\frac{x}{5+x}=t$, so that $\frac{dx}{(5+x)^2}=\frac{dt}{5}$ and the integral becomes

$\frac{1}{(5)^{11/2}}\int_0^{1/6}(t)^{7/2}(1-6t)^{9/2}dt$ and now from here do the substitution $6t=u$ and we simply obtain

$I_2=\frac{1}{5^{9/2}\times 6^{7/2}}I_1$ and we conclude $a=30$.

**47.(3)** $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=30$

$|abc\sin\theta\cos\phi|=30\Rightarrow\theta=\frac{\pi}{2},\phi=0\Rightarrow\vec{a},\vec{b},\vec{c}$ -are mutually perpendicular

$$\left(2\vec{a}+\vec{b}+\vec{c}\right).\left[\left(\vec{a}\times\vec{c}\right)\times\left(\vec{a}-\vec{c}\right)+\vec{b}\right]=\left(2\vec{a}+\vec{b}+\vec{c}\right).\left[\left(\vec{a}.\vec{a}\right)\vec{c}+c^2.\vec{a}+\vec{b}\right]$$

$$=2a^2c^2+b^2+a^2c^2=3a^2c^2+b^2=300+9=309$$

$$\therefore\frac{k}{103}=\frac{309}{103}=3$$

**48.(25)** A : Mr. A reaches late

$B_1$ : A goes to school by walking

$B_2$ : A takes bus to school

E: A will be on time for atleast one out of 2 consecutive days.

$P(B_1)=\frac{3}{4};\ P(B_2)=\frac{1}{4};\ P(A/B_1)=\frac{1}{3}$

$P(A/B_2)=\frac{2}{3}$

$P(A)=P(B_1\cap A)+P(B_2\cap A)=\frac{3}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{2}{3}=\frac{5}{12}$

$P(E)=1-P(A\cap A)=1-\frac{5}{12}\times\frac{5}{12}=\frac{119}{144}\equiv\frac{p}{q}$

$q-p=144-119=25$

**49.(190)** $B^2=I$

$$AB=\begin{bmatrix} a & x & p \\ y & q & b \\ r & c & z \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}=\begin{bmatrix} p & x & a \\ b & q & y \\ z & c & r \end{bmatrix}$$

$$AB=AB^3=..........=AB^{19}=\begin{bmatrix} p & x & a \\ b & q & y \\ z & c & r \end{bmatrix}$$

tr. $\left(AB+AB^3+.....+AB^{19}\right)=210$

$10(p+q+r)=210\Rightarrow p+q+r=21, p,q,r\in N$

$p'+q'+r'=18,\ p',q',r'\in W$

Number of ordered triplets $(p,q,r)={}^{20}C_2=\frac{20\times 19}{2}=190$

**50. (41)** Let $P(E_1)=a, P(E_2)=b$ and $P(E_3)=c$

$3a(1-b)(1-c)=(1-a)b(1-c)=9(1-a)(1-b)c=3(1-a)(1-b)(1-c)$

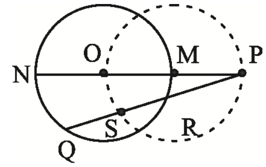
$\frac{3a}{1-a}=\frac{b}{1-b}=\frac{9c}{1-c}=3\Rightarrow a=\frac{1}{2},b=\frac{3}{4},c=\frac{1}{4}$

Now, $\begin{vmatrix} 1/2 & 3/4 & 1/4 \\ 3/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{vmatrix}=\frac{1}{64}\begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}=\frac{-9}{32};$ $\quad\frac{a}{b}=\frac{9}{32}\Rightarrow a+b=41$

**51.(C)**

**52.(B)**

I. Locus of S is a part of circle with OP as diameter passing inside the circle C.



II.(D) $(PR)(PQ) = (NP)(MP) = (d+r)(d-r) = d^2 - r^2$

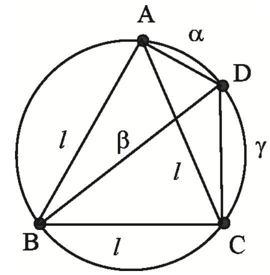
$= (PS - SR)(PS + SQ) = PS^2 - SQ^2$

$= (PS)^2 - (SQ)(SR)$

III.(A) Using Ptolemy's theorem

$(BD)(AC) = (AB)(CD) + (BC)(AD)$

$\beta l = l\gamma + \alpha l \Rightarrow \beta = \gamma + \alpha$




**53.(D)**

**54.(B)**

**I.** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 2 \Rightarrow \alpha = \beta$

$D = 0 \Rightarrow 64 - 4(k^2 - 6k) = 0$

$k^2 - 6k - 16 = 0 \Rightarrow (k-8)(k+2) = 0$

**II.** $(k-2)(3k+8) < 0$

$-\frac{8}{3} < k < 2$

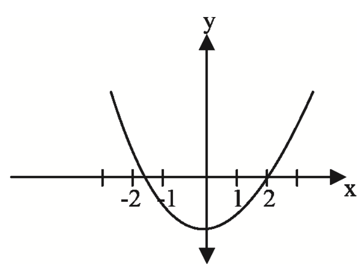
**III.** $|\alpha - \beta| < \sqrt{3}$

$\frac{\sqrt{4k^2 - 16}}{4} < \sqrt{3} \Rightarrow \sqrt{k^2 - 4} < 2\sqrt{3}$

$0 \le k^2 - 4 < 12$

$k \in (-\sqrt{12}, -2) \cup (2, \sqrt{12})$

**IV.**




$(x-2)(2kx+5) = 0$ where $k > 0$

$-2 \le \frac{-5}{2k} < -1 \Rightarrow 2 \ge \frac{5}{2k} > 1$

$\frac{5}{4} \le k < \frac{5}{2}$